Lecture 22

- The Generalized Pigeonhole Principle: If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects

- "What is the probability that at least 2 students in his classroom have the same birthday?"
  - ↳ we have 96 people
  - ↳ Assume 365 days in year.

$$P_r[E] = \frac{|E|}{|u|} = \frac{365 \times 364 \times \dots \times (365-96+1)}{365^{96}} = \frac{P(365,96)}{365^{96}} = \frac{1}{920\,000}$$

↑ this is assuming birthday distribution is uniform.

- Can also write $P_r[E] = \frac{365 \times 364 \times \cdots \times (365-96+1)}{365 \times 365 \times \cdots \times 365} = \left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)\cdots\left(\frac{365-96+1}{365}\right)$

probability that 1st birthday does not have overlap

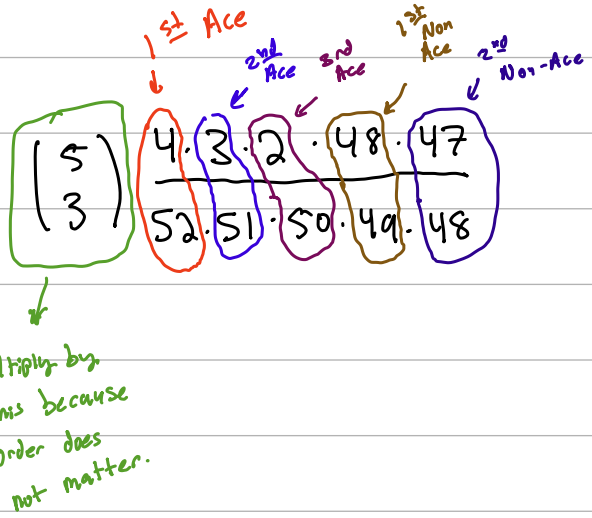
probability that 3rd birthday does NOT have overlap

Probability of an Event:

- Dealer gives you 5 cards out of 52, and each card has $\frac{1}{52}$ chance (initially)
- What is probability that you will get 3 Aces when dealer randomly gives you 5 cards?

↳ The first three cards are set to aces.
↳ so now you have 49 cards left.
↳ you have 2 cards left to choose
  ↳ none can be aces
  ↳ so you have 1 ace left in deck
  ↳ take it out, now you have 48 cards
  ↳ so choose $\binom{48}{2}$
↳ final answer $\frac{\binom{4}{3}\cdot\binom{48}{2}}{\binom{52}{5}}$

Or



- Suppose Machine dealing 5 cards
  - ↳ What is chance of getting 4 Aces?

$$\binom{5}{4}\left[\frac{4}{52} \times \frac{3}{52} \times \frac{2}{52} \times \frac{1}{52}\right]$$

- What if you have unlimited cards
  - ↳ machine gives Ace 10% of the time
  - ↳ machine gives Non-Ace 90% of the time.

$$\binom{5}{4}\left[\underbrace{\frac{1}{10}}_{A} \times \underbrace{\frac{1}{10}}_{A} \times \underbrace{\frac{1}{10}}_{A} \times \underbrace{\frac{1}{10}}_{A} \times \underbrace{\frac{9}{10}}_{*}\right]$$

probability of sequence AAAA*

but we have $\binom{5}{4}$ to get 4 aces (different orders)

- Stats show newborns are 51% male and 49% female.

- If a couple has 2 kids, what is probability that they have at least girl.

100% - percentage of 2 boys

$1 - \left(\frac{51}{100}\right)\left(\frac{51}{100}\right)$ = answer

or Probability (1 girl) + Probability (exactly 2 girls)

$\binom{2}{1}(0.49)(0.51) + \binom{2}{2}(0.49)(0.49)$